



## Insight — Application Note 3.08

### Ion Viscosity and Loss Factor

#### Permittivity and loss factor

Dielectric permittivity  $\epsilon^*$  is a quantity with real and imaginary parts:

$$(Eq. 8-1) \quad \epsilon^* = \epsilon_0 (\epsilon' + i \epsilon'') = \epsilon_0 [\epsilon' + i \sigma / (\epsilon_0 \omega)]$$

Where:  $\epsilon_0 = 8.854 \times 10^{-14}$  F/cm (permittivity of free space)  
 $\omega = 2\pi \cdot f$  (angular frequency)  
 $f$  = oscillation frequency (Hz)

Conductivity ( $\sigma$ ) is the sum of frequency independent ( $\sigma_{DC}$ ) and frequency dependent ( $\sigma_{AC}$ ) components, as expressed below:

$$(Eq. 8-2) \quad \sigma = \sigma_{DC} + \sigma_{AC}$$

The real part of relative permittivity or relative dielectric constant ( $\epsilon'$ ) also has frequency independent and frequency dependent components, but they will not be treated in this chapter.

In an oscillating electric field,  $\sigma_{DC}$  arises from the flow of mobile ions while  $\sigma_{AC}$  arises from the rotation of stationary dipoles. The dimensionless term *loss factor* ( $\epsilon''$ ) is a measure of the dissipation, or loss, of electromagnetic energy as heat and is given by:

$$(Eq. 8-3) \quad \epsilon'' = \sigma / (\omega \epsilon_0) = (\sigma_{DC} + \sigma_{AC}) / (\omega \epsilon_0)$$

During the early part of cure, when a thermoset is most conductive,  $\sigma_{DC}$  tends to dominate the dielectric response across a broad range of frequencies. At sufficiently low frequencies  $\sigma_{DC}$  may also be the significant component through the entire cure. When  $\sigma_{DC}$  dominates cure behavior,  $\sigma_{AC}$  is insignificant and loss factor may be approximated as:

$$(Eq. 8-4) \quad \epsilon'' \approx \sigma_{DC} / (\omega \epsilon_0)$$

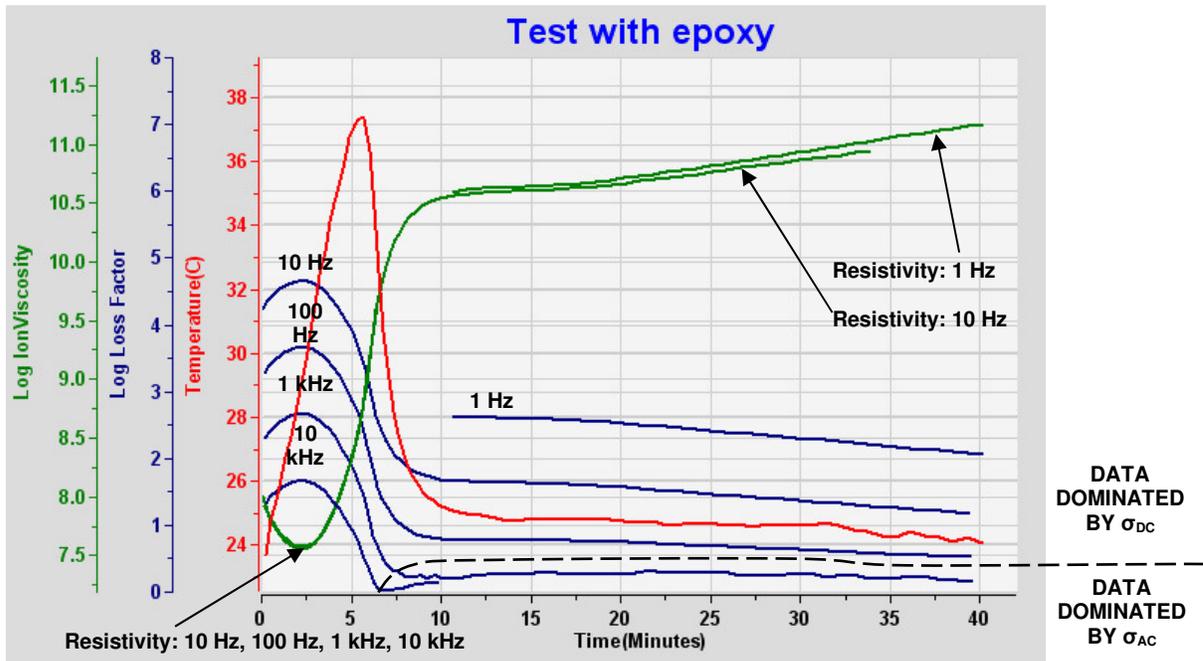
In this case, loss factor is inversely proportional to frequency. For example, if the excitation frequency ( $\omega$ ) *decreases* by a factor of 10, loss factor *increases* by the

same factor of 10—this relationship identifies when  $\sigma_{DC}$  dominates the dielectric response and can indicate cure state.

During the latter part of cure, frequency *dependent* conductivity due to dipoles may dominate the dielectric response, especially at higher frequencies. It is important to identify these times and frequencies to avoid misinterpreting data when studying cure.

### Cure monitoring with multiple frequencies

Figure 8-1 shows dielectric data at multiple frequencies during the cure of “five-minute epoxy.” A plot of loss factor can reveal when frequency independent conductivity  $\sigma_{DC}$  dominates the dielectric response. In the early part of this cure, loss factors for 10 Hz, 100 Hz, 1 kHz, and 10 kHz all are inversely proportional to frequency.



**Figure 8-1**  
**Multiple frequency data from cure of five-minute epoxy**

In the latter part, however, only loss factors for 1 Hz, 10 Hz and 100 Hz are inversely proportional to frequency.

Resistivity  $\rho$  is the inverse of conductivity, as expressed in equation 8-5:

(Eq. 8-5) 
$$\rho = 1 / (\sigma_{DC} + \sigma_{AC}) = 1/(\omega \epsilon_0 \epsilon'')$$

When frequency independent conductivity  $\sigma_{DC}$  dominates the data, resistivity is also largely frequency independent. Frequency independent resistivity  $\rho_{DC}$  is called *ion viscosity*, and is characterized by the overlap of curves at multiple frequencies, as shown in Figure 8-1.

During thermoset cure, the change in ion viscosity is typically proportional to the change in mechanical viscosity until they diverge around the time of gelation (See Figure 7-4). Even after mechanical viscosity becomes immeasurable, however, crosslinking continues and the growing polymer network still presents greater and greater resistance to the flow of ions. Consequently, frequency independent resistivity  $\rho_{DC}$ —ion viscosity—can be used for determining cure state through the entire cure.



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