Insight — Application Note 3.01

Terms and Definitions

$e = 2.71828$	Base of natural logarithm
$i = \sqrt{-1}$	Square root of -1
$\pi = 3.14159$	Pi
$\varepsilon_0 = 8.854 \times 10^{-14}$ F/cm	Permittivity of free space
$k = 8.61733 \times 10^{-5}$ eV/K	Boltzmann's constant
$q = 1.602 \times 10^{-19}$ C	Magnitude of electronic charge
f	Frequency (cycles/s or Hz)
$\omega = 2\pi f$	Angular frequency (radians/s)
q	Charge (coulomb)
t	Time (s)
σ	Conductivity ($\text{ohm}^{-1} - \text{cm}^{-1}$)
$\rho = 1/\sigma$	Resistivity, or ion viscosity (ohm-cm)
ε	Permittivity (F/cm)
$\varepsilon^* = \varepsilon_0 (\varepsilon' - i\varepsilon'')$	Permittivity as a complex quantity (F/cm)
$\kappa = \varepsilon' - i\varepsilon''$	Relative permittivity* or dielectric constant (unitless)
$\varepsilon' = \mathrm{Re}(\varepsilon^*/\varepsilon_0)$	Relative permittivity* or dielectric constant (unitless)
$\varepsilon'' = \mathrm{Im}(\varepsilon^*/\varepsilon_0) = \sigma / \omega\varepsilon_0$	Loss factor or loss index (unitless)
$D = \tan\delta = \varepsilon''/\varepsilon'$	Dissipation factor or loss tangent (unitless)
$\delta = \tan^{-1}(\varepsilon''/\varepsilon')$	Delta (unitless)
$\delta \sim \varepsilon''/\varepsilon'$	Delta for small $\varepsilon''/\varepsilon'$ (unitless)
A/D	Ratio of electrode area to distance between parallel plate electrodes (cm)
V	Voltage (volt)
$I = q/t$	Current (amp)
$C = q/V$	Capacitance (farad)
$G = I/V$	Conductance (ohm^{-1})
$R = V/I = 1/G$	Resistance (ohm)

* There is inconsistency in use of the term *relative permittivity*. In some literature it is the complex quantity $\varepsilon/\varepsilon_0 = (\varepsilon' - i\varepsilon'')/\varepsilon_0$ and in others it is the real part of permittivity $\mathrm{Re}(\varepsilon/\varepsilon_0) = \varepsilon'$.

$C = \varepsilon_0\, \varepsilon' A/D$	Capacitance (farad)
$G = \sigma A/D$	Conductance (siemens or ohm^{-1})
$R = \rho /(A/D)$	Resistance (ohm)
$Y = G + i\omega C$	Admittance (siemens or ohm^{-1})
$Z = 1/Y = 1/(G + i\omega C)$	Impedance (ohm)
α	Degree of cure (unitless, between 0 and 1)
α_{max}	Maximum degree of cure at cure temperature
M_w	Molecular weight (gm/mole)
M_{w0}	Molecular weight at α = 0 (uncured)
$M_{w\infty}$	Molecular weight at α = 1 (fully cured)
T_{Cure}	Cure or process temperature (K or °C)
T_g	Glass transition temperature (K or °C)

ε' (free space)	= 1.0	Relative permittivity of free space
ε' (air)	= 1.0	Relative permittivity of air
ε' (teflon)	~ 2.2	Relative permittivity of teflon
ε' (mineral oil)	~ 2.2	Relative permittivity of mineral oil
ε' (polyimide)	~ 3.6	Relative permittivity of polyimide
ε' (alumina)	~ 9.8	Relative permittivity of alumina

DiBenedetto T_g model:

$$\frac{(T_g - T_{g0})}{(T_{g\infty} - T_{g0})} = \frac{\lambda\, \alpha}{(1 - (1 - \lambda)\, \alpha)}$$

Where:
T_g = Glass transition temperature (K or °C)
T_{g0} = Glass transition temperature at α = 0 (uncured)
T_{gmax} = Maximum glass transition at T_{Cure}
$T_{g\infty}$ = Glass transition temperature at α = 1 (fully cured)
λ = Adjustable parameter

Debye relaxation for relative permittivity and loss factor:

$$\varepsilon' = \varepsilon'_u + \frac{\varepsilon'_r - \varepsilon'_u}{1 + (\omega\tau)^2} \quad , \quad \varepsilon'' = \sigma / \omega\varepsilon_0 + (\varepsilon'_r - \varepsilon'_u)\frac{\omega\tau}{1 + (\omega\tau)^2}$$

Where:
ε'_u = Unrelaxed (high frequency) relative permittivity
ε'_r = Relaxed (low frequency) relative permittivity
ω = $2\pi f$ (radians/s)
τ = Dipole relaxation time (s)

Havriliak-Negami relaxation for relative permittivity and loss factor:

$$\varepsilon' = \varepsilon'_u + (\varepsilon'_r - \varepsilon'_u)\,(1 + 2(\omega\tau)^{\alpha}\cos(\pi\alpha/2) + (\omega\tau)^{2\alpha})^{-\beta/2}\cos(\beta\theta)$$

$$\varepsilon'' = (\varepsilon'_r - \varepsilon'_u)\,(1 + 2(\omega\tau)^{\alpha}\cos(\pi\alpha/2) + (\omega\tau)^{2\alpha})^{-\beta/2}\sin(\beta\theta)$$

Where:
$\theta = \tan^{-1}[(\omega\tau)^{\alpha}\sin(\pi\alpha/2) / (1 + (\omega\tau)^{\alpha}\cos(\pi\alpha/2))]$
α = Empirical “broadness” parameter
β = Empirical “asymmetry” parameter
ε'_u = Unrelaxed (high frequency) relative permittivity
ε'_r = Relaxed (low frequency) relative permittivity
ω = $2\pi f$ (radians/s)
τ = Dipole relaxation time (s)

Apparent relative permittivity and loss factor due to electrode polarization:

$$\varepsilon'_{x} = \varepsilon' \, (D / 2t_{b}) \frac{(\varepsilon''/\varepsilon')^{2} + (D / 2t_{b})}{(\varepsilon''/\varepsilon')^{2} + (D / 2t_{b})^{2})}$$ Apparent relative permittivity

$$\varepsilon''_{x} = \varepsilon'' \, (D / 2t_{b}) \frac{(D / 2t_{b}) - 1}{(\varepsilon''/\varepsilon')^{2} + (D / 2t_{b})^{2})}$$ Apparent loss factor

Where:
t_{b} = boundary layer thickness
D = distance between electrodes or plate separation
ε'_{x} = uncorrected permittivity
ε''_{x} = uncorrected loss factor
ε' = actual permittivity
ε'' = actual loss factor

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